Flow in pipes

A lecture by
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Pipe flow in water supply

Pipe flow in irrigation

Pipe flow in dams
Pipe flow in dams

Pipes and pumps

Pipes over rivers

Pipes in industrial setups
Laminar & Turbulent Flows

- Laminar flow: orderly, in layers
- Turbulent flow: disorderly, eddies
- Transitional: intermittently turbulent
- Criteria: Reynolds number
- Critical Re = 2000

Flow films

- LaminarTurbulentFaucetFlow.MOV
- LaminarPipeFlow.MOV
- TurbulentPipeFlow.MOV
- TransitionalPipeFlow.MOV
- LaminarTurbulentCombo.MOV
- TurbulentFlowAroundUs.MOV

Laminar & turbulent flow out of a faucet

Laminar flow in a pipe
Turbulent flow in a pipe

Transitional flow in a pipe

Laminar and turbulent flow visualization

Hydraulic Radius - 1

- A = cross-sectional area
- P = wetted perimeter (length of boundary in contact with water)

- Hydraulic Radius: $R_h = \frac{A}{P}$
Hydraulic Radius - 2

• FOR PIPE FLOW
• \( A = \pi D^2/4, \quad P = \pi D^2, \quad R_h = A/P = D/4 \)

Analysis of motion of pipe flow

Head losses and shear stress

\[ h_f = \frac{\tau_o \cdot L}{\gamma \cdot R_h} \]

Applies to:
• Any x-section
• Laminar or turbulent flow
Smooth and rough walls

- Examples:
  - Glass
  - Plexiglass
  - PVC

- Examples:
  - Copper
  - Concrete

Rough walls characterized by an absolute roughness ($e$)

Dimensional Analysis of Pipe Flow -1

Dimensional Analysis of Pipe Flow -2

Head Losses in Pipe Flow

\[
\overline{h} = \frac{C_f g V^2}{2}
\]

Replace this result with:

\[
\overline{h} = \frac{C_f g V^2}{2} = \frac{C_f g V^2}{2 \frac{R_{th}}{g}} = \frac{C_f g V^2}{2 \frac{R_{th}}{g}}
\]

Slope of the energy line:

\[
S = \frac{h_f}{L} = \frac{C_f g V^2}{2 \frac{R_{th}}{g}} = \text{Energy gradient}
\]

\[
C_f = \text{friction coefficient}
\]
Head losses in circular conduits

- Start from
  \[ h_f = C_f \left( \frac{L}{R_h} \right) \frac{V^2}{2g} \]
- For a circular pipe, \( R_h = D/4 \)
- Replace \( C_f = f/4 \)
- Darcy-Weisbach equation:
  \[ h_f = f \left( \frac{L}{D} \right) \frac{V^2}{2g} \]
- \( f = \text{Darcy-Weisbach friction factor} \)
- \( C_f = 4f \) = Fanning friction factor (used in gas flow)

Analysis of motion of pipe flow

- Head losses and shear stress
- \( h_f = \text{head loss in length } L \)
- \( \tau_o = \text{wall shear stress} \)
- \( \gamma = \text{specific weight} \)
- \( R_h = \text{hydraulic radius} \)

\[ h_f = \frac{\tau_o \cdot L}{\gamma \cdot R_h} \]

- Applies to:
  - Any x-section
  - Laminar or turbulent flow

Shear stress distribution

- Equilibrium of forces on an element of radius \( r \), steady flow.

\[ \sum F_x = ma_x, a_x = 0 \]
Shear stress – linear distribution

- From an earlier result:
  \[ h_f = \frac{\tau_o \cdot L}{\gamma \cdot R_h} \]

- Also,
  \[ h_f = \frac{2\tau(r)L}{\gamma \cdot r} \]

- With \( R_h = \frac{D}{4} = \frac{r_o}{2} \)

\[ \tau(r) = \tau_o \cdot \frac{r}{r_o} \]

Wall shear stress and friction factor

- Combine the result
  \[ h_f = \frac{\tau_o \cdot L}{\gamma \cdot R_h} = \frac{\tau_o \cdot L}{\gamma \cdot (D/4)} \]

- With Darcy-Weisbach
  \[ h_f = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g} \]

  \[ \tau_o = \frac{f}{8} \cdot \rho \cdot V^2 \]

Friction in non-circular conduits

- Use \( D = 4R_h \) in Darcy-Weisbach
  \[ h_f = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g} = f \cdot \frac{L}{4R_h} \cdot \frac{V^2}{2g} \]

- With
  \[ \text{Re} = \frac{\rho VD}{\mu} = \frac{\rho V \cdot 4R_h}{\mu} \]

Laminar flow in a circular pipe
Velocity and shear stress distributions

Centerline and mean velocities – discharge and head losses

Hagen-Poiseuille law for laminar flow

\[ h_f = \frac{32 \nu L V}{gD^2} \]

1. \( h_f \sim V \)
2. Equation involves no empirical coefficient
3. Equation involves only fluid properties, \( g \), and \( V \)

Combining Hagen-Poiseuille with Darcy-Weisbach

\[ f = \frac{64 \nu L V}{gD^2} \]

with

\[ Re = \frac{VD}{\nu} = \frac{8VD}{\mu} \]

then, for laminar flow

\[ f = \frac{64}{Re} \text{ for } |Re| < 2000 \]
Exercises - 1

Exercises - 2

Exercises - 3

- Rounded entrance produces uniform flow (inviscid flow behavior)
- Velocities at the wall are zero (no-slip condition)
- A viscous boundary layer develops, but an inviscid core remains before flow is fully developed
Exercise in flow development

Exercise 8.8.2. In Exer. 8.5.4 what will be the approximate distance from the pipe entrance to the first point at which the flow is established?

In Exer. 8.5.4. we found \( R_e = 21.45 \), \( D = 0.10 \text{ m} \), then

\[
L_e = 0.058 R_e D = 0.058 \times 21.45 \times 0.10 \text{ m}
\]

\[
L_e = 0.124 \text{ m} = 124 \text{ mm}
\]